EXERCISE–12

Part : (A) Only one correct option

1. The eccentricity of the ellipse $4x^2 + 9y^2 + 8x + 36y + 4 = 0$ is

(A) $\frac{5}{6}$ (B) $\frac{3}{5}$ (C) $\frac{\sqrt{2}}{3}$ (D) $\frac{\sqrt{5}}{3}$

2. The equation of the ellipse with its centre at (1, 2), focus at (6, 2) and passing through the point (4, 6) is

(A) $\frac{(x-1)^2}{45}+\frac{(y-2)^2}{20}=1$ (B) $\frac{(x-1)^2}{20}+\frac{(y-2)^2}{45}=1$

(C) $\frac{(x-1)^2}{25}+\frac{(y-2)^2}{16}=1$ (D) $\frac{(x-1)^2}{16}+\frac{(y-2)^2}{25}=1$

3. The eccentricity of the ellipse which meets the straight line $\frac{x}{7}+\frac{y}{2}=1$ on the axis of x and the straight line $\frac{x}{3}-\frac{y}{5}=1$ on the axis of y and whose axes lie along the axes of coordinates, is

(A) $\frac{3\sqrt{2}}{7}$ (B) $\frac{2\sqrt{6}}{7}$ (C) $\frac{\sqrt{3}}{7}$ (D) none of these

4. The curve represented by x = 3 (cos t + sin t), y = 4 (cos t – sin t), is
(A) ellipse (B) parabola (C) hyperbola (D) circle

5. Minimum area of the triangle by any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with the coordinate axes is

(A) $\frac{a^2+b^2}{2}$ (B) $\frac{(a+b)^2}{2}$ (C) ab (D) $\frac{(a-b)^2}{2}$

6. A circle has the same centre as an ellipse & passes through the focii F_1 & F_2 of the ellipse, such that the two curves intersect in 4 points. Let 'P' be any one of their point of intersection. If the major axis of the ellipse is 17 & the area of the triangle PF_1F_2 is 30, then the distance between the focii is :

(A) 11 (B) 12 (C) 13 (D) 15

7. Q is a point on the auxiliary circle corresponding to the point P of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. If T is the foot of the perpendicular dropped from the focus S onto the tangent to the auxiliaryy circle at Q then the Δ SPT is :
(A) isosceles (B) equilateral (C) right angled (D) right isosceles

8. $x - 2y + 4 = 0$ is a common tangent to $y^2 = 4x$ & $\frac{x^2}{4}+\frac{y^2}{b^2}=1$. Then the value of 'b' and the other common tangent are given by :

(A) $b=\sqrt{3}$; x + 2y + 4 = 0 (B) b = 3; x + 2y + 4 = 0

(C) $b=\sqrt{3}$; x + 2y – 4 = 0 (D) $b=\sqrt{3}$; x – 2y – 4 = 0

9. The locus of point of intersection of tangents to an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the points whose the sum of eccentric angles is constant, is :
(A) a hyperbola (B) an ellipse (C) a circle (D) a straight line

10. A tangent having slope of $-\frac{4}{3}$ to the ellipse $\frac{x^2}{18}+\frac{y^2}{32}=1$ intersects the major & minor axes in points A & B respectively. If C is the centre of the ellipse, then the area of the triangle ABC is :
(A) 12 sq. units (B) 24 sq. units (C) 36 sq. units (D) 48 sq. units

11. The normal at a variable point P on an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ of eccentricity 'e' meets the axes of the ellipse in Q and R then the locus of the mid-point of QR is a conic with an eccentricity e' such that :

(A) e' is independent of e (B) $e' = 1$
(C) $e' = e$ (D) $e' = 1/e$

12. y = mx + c is a normal to the ellipse, $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ = 1, if c^2 is equal to :

(A) $\frac{(a^2-b^2)^2}{a^2m^2+b^2}$ (B) $\frac{(a^2-b^2)^2}{a^2m^2}$ (C) $\frac{(a^2-b^2)^2m^2}{a^2+b^2m^2}$ (D) $\frac{(a^2-b^2)^2m^2}{a^2m^2+b^2}$

13. An arc of a bridge is semi-elliptical with major axis horizontal. The length of the base is 9 meter and the highest part of the bridge is 3 meter from the horizontal. The best approximation of the Pillar 2 meter from the centre of the base is :
(A) 11/4 m (B) 8/3 m (C) 7/2 m (D) 2 m

14. Point 'O' is the centre of the ellipse with major axis AB & minor axis CD. Point F is one focus of the ellipse. If OF = 6 & the diameter of the inscribed circle of triangle OCF is 2, then the product (AB) (CD) is
(A) 64 (B) 12 (C) 65 (D) 3

15. An ellipse is such that the length of the latus rectum is equal to the sum of the lengths of its semi principal axes. Then:
(A) Ellipse bulges to a circle
(B) Ellipse becomes a line segment between the two foci
(C) Ellipse becomes a parabola
(D) none of these

16. A line of fixed length (a + b) moves so that its ends are always on two fixed perpendicular straight lines. The locus of the point which divided this line into portions of lengths a & b is:
(A) an ellipse (B) an hyperbola (C) a circle (D) none of these

17. The line 2x + y = 3 cuts the ellipse $4x^2 + y^2 = 5$ at P and Q. If θ be the angle between the normals at these points, then tanθ =
(A) 1/2 (B) 3/4 (C) 3/5 (D) 5

18. The focal chord of $y^2 = 16x$ is tangent to $(x-6)^2 + y^2 = 2$, then the possible values of the slope of this chord are **[IIT – 2003]**

(A) {– 1, 1} (B) {– 2, 2} (C) $\left\{-2,\frac{1}{2}\right\}$ (D) $\left\{2,-\frac{1}{2}\right\}$

19. A tangent is drawn to ellipse $x^2 + 2y^2 = 2$. Then the locus of mid point of portion of the tangent intercepted between coordinate axes. **[IIT - 2004]**

(A) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ (B) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$ (C) $\frac{x^2}{2}+\frac{y^2}{4}=1$ (D) $\frac{x^2}{4}+\frac{y^2}{2}=1$

20. The locus of mid point of the intercept of the tangent drawn from an external point to the ellipse $x^2 + 2y^2 = 2$ between the coordinate axes, is **[IIT - 2004]**

(A) $\frac{1}{x^2}+\frac{1}{2y^2}=1$ (B) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$ (C) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ (D) $\frac{1}{2x^2}+\frac{1}{y^2}=1$

21. An ellipse has OB as semi-minor axis, F and F′ its foci and the angle FBF′ is a right angle. Then, the eccentricity of the ellipse is **[IIT - 2005]**

(A) $\frac{1}{4}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{1}{2}$

Part : (B) May have more than one options correct

22. The tangent at any point 'P' on the standard ellipse with focii as S & S′ meets the tangents at the vertices A & A′ in the points V & V′, then :
(A) (AV) (A′ V′) = b^2 (B) (AV) (A′ V′) = a^2
(C) ∠V′ SV = 90º (D) V′ S′ VS is a cyclic quadrilateral

23. Identify the statements which are True.
(A) the equation of the director circle of the ellipse, $5x^2 + 9y^2 = 45$ is $x^2 + y^2 = 14$.
(B) the sum of the focal distances of the point (0, 6) on the ellipse $\frac{x^2}{25}+\frac{y^2}{36}$ = 1 is 10.
(C) the point of intersection of any tangent to a parabola & the perpendicular to it from the focus lies on the tangent at the vertex.
(D) the line through focus and $(at_1^2, 2at_1)$ on $y^2 = 4ax$, meets it again in the point $(at_2^2, 2at_2)$ iff $t_1t_2 = -1$.

24. The Cartesian equation of the curve whose parametric equation is x = 2t – 3 and y = $4t^2$ – 1 is given by
(A) $(x + 3)^2 - y - 1 = 0$ (B) $x^2 + 6x - y + 8 = 0$
(C) $(y + 1)^2 + x + 3 = 0$ (D) $y^2 + 6x - 2y + 4 = 0$

25. If P is a point of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, whose focii are S and S′. Let ∠PSS′ = α and ∠PS′S = β, then
(A) PS + PS′ = 2a, if a > b
(B) PS + PS′ = 2b, if a < b
(C) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{1-e}{1+e}$
(D) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{\sqrt{a^2-b^2}}{b^2}\,[a - \sqrt{a^2-b^2}\,]$ when a > b

26. If the distance between the focii of an ellipse is equal to the length of its latus rectum, the eccentricity of the ellipse is :
(A) $\frac{\sqrt{5}+1}{2}$ (B) $\frac{\sqrt{5}-1}{2}$ (C) $\frac{\sqrt{5}-2}{2}$ (D) $\frac{2}{\sqrt{5}+1}$

EXERCISE–13

1. Let use consider an ellipse whose major and minor axis are 3x + 4y – 7 = 0 and 4x – 3y – 1 = 0 respectively 'P' be a variable point on the ellipse at any instance, it is given that distance of 'P' from major and minor axis are 4 and 5 respectively. It is also given that maximum distance of 'P' from minor axis is $5\sqrt{2}$, then find its eccentricity.

2. Prove that the area of the triangle formed by the three points on an ellipse, whose eccentric angle are θ, φ, and ψ, is $2\,ab\,\sin\frac{\phi-\psi}{2}\sin\frac{\psi-\theta}{2}\sin\frac{\theta-\phi}{2}$.

3. Find the equation of tangents to the ellipse $\frac{x^2}{50} + \frac{y^2}{32} = 1$ which passes through a point (15, – 4).

4. If 'P' be a moving point on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ in such a way that tangent at 'P' intersect x = $\frac{25}{3}$ at Q then circle on PQ as diameter passes through a fixed point. Find that fixed point.

5. Any tangent to an ellipse is cut by the tangents at the ends of major axis in the points T and T′. Prove that the circle, whose diameter is T T′ will pass through the foci of the ellipse.

6. If 3x + 4y = 12 intersect the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ at P and Q, then find the point of intersection of tangents at P and Q.

7. Find the equation of the largest circle with centre (1, 0) that can be inscribed in the ellipse $x^2 + 4y^2 = 16$.

8. If P is a variable point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose foci are S and S′, then prove that the locus of the incentre of ΔPSS′ is an ellipse whose eccentricity is $\sqrt{\frac{2e}{1+e'}}$ where e is the eccentricity of the given ellipse.

9. The tangent at a point P (a cosθ, b sinθ) of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, meets its auxiliary circle in two points, the chord joining which subtends a right angle at the centre. Show that the eccentricity of the ellipse is $(1 + \sin^2\theta)^{-1/2}$.

10. A circle of radius r is concentric with the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Prove that the common tangent is inclined to the major axis at an angle $\tan^{-1}\sqrt{\left(\frac{r^2-b^2}{a^2-r^2}\right)}$.

11. ‘O’ is the origin & also the centre of two concentric circles having radii of the inner & the outer circle as ‘a’ & ‘b’ respectively. A line OPQ is drawn to cut the inner circle in P & the outer circle in Q. PR is drawn parallel to the y– axis & QR is drawn parallel to the x– axis. Prove that the locus of R is an ellipse touching the two circles. If the focii of this ellipse lie on the inner circle, find the ratio of inner : outer radii & find also the eccentricity of the ellipse.

12. If any two chords be drawn through two points on the major axis of an ellipse equidistant from the centre, show that $\tan\frac{\alpha}{2}.\tan\frac{\beta}{2}.\tan\frac{\gamma}{2}.\tan\frac{\delta}{2}=1$, where $\alpha, \beta, \gamma, \delta$ are the eccentric angles of the extremities of the chords.

13. The tangent at a point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intersects the major axis in T & N is the foot of the perpendicular from P to the same axis. Show that the circle on NT as diameter intersects the auxiliary circle orthogonally.

14. Show that the equation of the tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at the points of intersection with the line, $px + qy + 1 = 0$ is $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)(p^2a^2 + q^2b^2 - 1) = (px + qy + 1)^2$.

15. Common tangents are drawn to the parabola $y^2 = 4x$ & the ellipse $3x^2 + 8y^2 = 48$ touching the parabola at A & B and the ellipse at C & D. Find the area of the quadrilateral.

16. A tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the ellipse $\frac{x^2}{a}+\frac{y^2}{b}=a+b$ in the points P and Q; prove that the tangents at P and Q are at right angles.

17. Let P be a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ for which the area of the ΔPON is the maximum where O is the origin and N is the foot of the perpendicular from O to the tangent at P. Find the maximum area and eccentric angle of point P.

18. Find the equation of the largest circle with centre (1, 0) that can be inscribed in the ellipse $x^2 + 4y^2 = 16$. **[IIT - 1999]**

19. Let P be point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $0 < b < a$. Let the line parallel to y–axis passing through P meet the circle $x^2 + y^2 = a^2$ at the point Q such that P and Q are on the same side of x–axis. For two positive real numbers r and s. Find the locus of the point R on PQ such that PR : RQ = r : s as P varies over the ellipse. **[IIT - 2001]**

20. Prove that in an ellipse, the perpendicular from a focus upon any tangent and the line joining the centre of the ellipse to the point of contact meet on the corresponding directrix. **[IIT - 2002]**

Answers

EXERCISE–12

1. D **2.** A **3.** B **4.** A **5.** C **6.** C **7.** A
8. A **9.** D **10.** B **11.** C **12.** C **13.** B **14.** C
15. A **16.** A **17.** B **18.** A **19.** A **20.** B **21.** C
22. ACD **23.** ACD **24.** AB **25.** ABD **26.** BD

EXERCISE–13

1. $\left(e=\frac{3}{5}\right)$ **3.** $4x + 5y = 40$, $4x - 35y = 200$.

4. (3, 0) **6.** $\left(\frac{25}{4},\frac{16}{3}\right)$ **11.** $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

18. $(x - 1)^2 + y^2 = \frac{11}{3}$ **19.** $\frac{x^2}{a^2}+\frac{y^2(r+s)^2}{(ra+sb)^2}=1$